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Private equity under Solvency II: Evidence from time series models

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EXECUTIVE SUMMARY

- Solvency II, an updated framework for determining the **regulatory capital requirements for European insurance companies,** is expected to **come into effect at the beginning of 2013.**

- The Solvency Capital Requirement (SCR) is based on a **Value-at-Risk** (VaR) measure calibrated to a **99.5% confidence level over a 1-year horizon** covering all risks that an insurer faces (insurance, market, credit, and operational risk).

- Under the standard formula, private equity investments are captured in the sub-module **'Other Equity'** for which a **stress factor** of **49%** is applied. This stress factor is calibrated by using the LPX 50 as underlying data. Partners Group **strongly disagrees** with using this index as a proxy for a portfolio of unlisted private equity investments.

- Partners Group advocates using an **index for unlisted private equity from Thomson Reuters** to calibrate the stress factor for unlisted private equity. In contrast to standard public market data, this data **exhibits auto-correlation**.

- Time series models allow analyzing data series with auto-correlation. Standard auto-regressive moving average processes (ARMA) **provide for a good fit** and allow calculating appropriate capital requirements.

- Calibrating a stress factor for private equity using time series models suggests a significantly smaller stress factor. Depending on the actual portfolio composition, data suggests a **stress factor of not more than 30%**, a **reduction of at least 40% (!)** when compared to the stress factor suggested by the standard model.

- Results on the aggregation parameter of private equity and **'Global Equity'** indicate that the **proposed parameter of 0.75 under the standard model is too high** and that a **coefficient of 0.6 provides a good approximation**. This should further increase the diversification benefits of private equity for an insurance company.

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DATA FOR UNLISTED PRIVATE EQUITY PORTFOLIO IS AVAILABLE

From the beginning of 2013, the existing Solvency I regime for European insurance companies will be replaced with Solvency II, an updated framework for determining the regulatory capital requirements. Under Solvency II, European insurance companies will need to calculate the Solvency Capital Requirement (SCR), which is based on a Value-at-Risk (VaR) measure calibrated to a 99.5% confidence level over a 1-year period[1]. The SCR is calculated by aggregating the SCR for various modules and sub-modules using a square root principle[2].

When calibrating the SCR for the various sub-modules, the European regulator has used a set of different time series. For example, for the sub-module 'Global Equity', the MSCI World was used to calculate a stress factor consistent with the stipulated 1-year 99.5% VaR[3]. Private equity investments were made part of a very diverse family of asset classes such as hedge funds, commodities and emerging markets equities. While it is understandable that a regulator will aim to simplify calculations under a standard formula, insurance companies which hold or plan to hold a significant exposure to private equity will aim to capture these investments on a more granular level. For the calibration of the capital requirement for private equity, the regulator suggested using the LPX 50. **We strongly oppose using the LPX 50 to calibrate a stress factor for private equity** as it does not capture the characteristics of an unlisted private equity portfolio. This has been extensively discussed in previous publications[4]. If the LPX 50 is however not appropriate, what are the alternatives?

There are data providers for unlisted (!) private equity return series. The longest time series is available from Thomson Reuters. The database contains pooled cash flow reports, capturing quarterly cash flows and valuations, which can be used as a proxy for the performance of the different segments of the market (e.g. the broad European private equity industry, or U.S. venture capital or buyout). We translate this information into return series by using a Mid-Point Dietz method, assuming that half of the cash flows are realized at the beginning of the $t+1$-th quarter and the other half at the end of the $t+1$-th quarter. In the following this data series is denoted as DQ:

$$DQ_{t+1} = \frac{NAV_{t+1} + CF_{t+1} - NAV_t}{NAV_t - \frac{1}{2}CF_{t+1}} \ .$$

While data is generally available from the mid-eighties, we use data from the time period starting at the beginning of 1990 up to the end of 2010 in order to have a sufficient number of funds in the sample and to avoid a major impact from immature investments ("J-curve"). Exhibit 1 shows the sample growth over the observed time period.

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[1] Results in this article are based on a recent study - *Private Equity unter Solvenz II, Irfan Ebibi, Swiss Federal Institute of Technology, Zürich* - supported by Partners Group. The authors would like to thank Prof. Embrechts and Prof. Wüthrich for their valuable input.

[2] Let $SCR_i \geq 0$ be capital requirements of submodules. The square-root principle is applied to calculate an overall capital requirement in the following way $SCR_{tot} = \sqrt{\sum_{i,j=1}^{n} \rho_{ij} SCR_i SCR_j}$, where the so-called aggregation coefficients ρ_{ij} have to be constrained such that the expression in the square root is positive, diversification benefits are considered adequately and the overall capital requirement corresponds to the overall 99.5%-VaR.

[3] See for example, CEIOPS' Advice for Level 2 Implementing Measures on Solvency II: Article 111 and 304, Equity risk sub-module, January 2010

[4] See *On the Suitability of the Calibration of Private Equity Risk in the Solvency II Standard Formula* (EDHEC Financial Analysis and Accounting Research Centre Publication, April 2010) and *Private equity allocations under Solvency II* (Partners Group research flash, August 2010)

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Exhibit 1: Development of sample size over time		
Sample	**Number of funds Q1 1990**	**Number of funds Q4 2010**
US buyout	98	553
European buyout	43	436
US venture capital	466	1'285
European venture capital	86	739
Total	**693**	**3'013**

Source: Thomson Reuters, Cash-flow summary report.

We argue **that this sample is representative enough to be used as an index** for private equity investment and that the data quality is sufficient. For the purpose of the analysis, we focus on what we believe is a typical private equity portfolio and analyze one combined time series, denoted as *pe,* by weighing the quarterly return series of the different regions and financing stages[5]. Hereafter we will only consider the returns of this diversified portfolio.

With Solvency II asking for a 1-year 99.5% VaR, we are mainly interested in the potential variation of annual private equity returns. Based on the quarterly Mid-Point Dietz method, we will approximate the yearly returns RY_t as follows:

$$(1 + RY_t) = (1 + DQ_{t-3})(1 + DQ_{t-2})(1 + DQ_{t-1})(1 + DQ_t).$$

In the following, statistical models are fitted based on log-returns with $X_t \equiv \log(1 + DQ_t)$. The yearly returns RY_t can now be written as

$$RY_t = \exp(X_{t-3} + X_{t-2} + X_{t-1} + X_t) - 1 .$$

Through these time series DQ_t and RY_t, private equity returns can be compared to a standard public market return series and analyzed using similar methods and metrics after a careful analysis of the data series properties (e.g. stationarity). One important characteristic where private equity returns typically differ from public market returns is the fact that quarterly private equity return series exhibit auto-correlation. Exhibit 2 shows that the auto-correlation of private equity returns is statistically significant for up to basically a one-year lag meaning that this quarter's return is linked to the returns observed during the last four quarters.

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[5] It is assumed that European and U.S. buyout are both allocated 40% and U.S. and European venture capital are both allocated 10% each. Depending on an insurance company's asset allocation, this can be adjusted accordingly.

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Exhibit 2: Auto-correlation function of quarterly private equity return series.
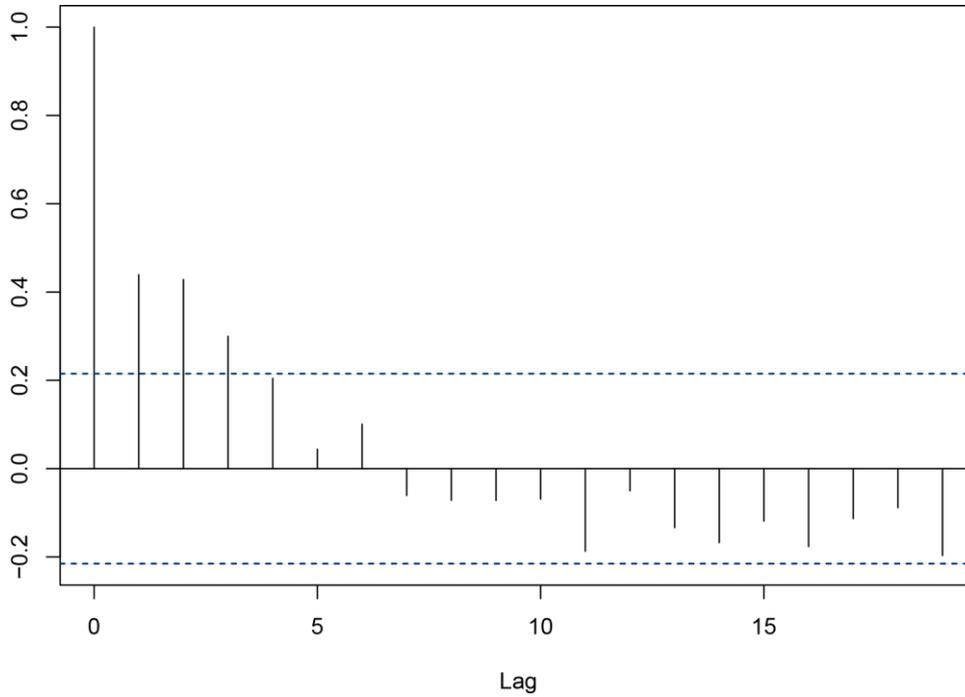
Source: Analysis based on Thomson Reuters data. The dashed lines show the 95%-confidence intervals.

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CALIBRATING A VALUE-AT-RISK USING TIME SERIES MODELS

Return series (in discrete time) of financial assets are often modeled using time series models. In its most simple form, a time series model can be the result of a person tossing a coin a hundred times. In more complex models, the outcome of one trial, its distribution ("how likely is each of the possible outcomes") as well as the dependence between different trials ("does the next trial depend on previous results") and the dimension ("how many coins do we flip in any one trial") may vary.

Given the fact that private equity return series exhibit auto-correlation, it is natural to use so-called autoregressive moving average, or short ARMA(p,q), processes for the analysis. ARMA processes are time series models where the realization at time t is (linearly) dependent on the past p realizations and a noise term, which itself depends linearly on the past q noise realizations of some chosen noise models[6]. ARMA models with $q = 0$ are denoted autoregressive models of order p or simply AR(p) models.

Standard statistical software packages allow estimating the parameters and the distribution of the noise process Z_t of ARMA(p,q) processes of a times series. Furthermore, in order to test whether the models provide a statistically good fit one can test the structure of the residuals. If the model fits well, the residuals should no longer exhibit auto-correlation. Exhibit 3 shows that by fitting an ARMA(1,1) model **autocorrelation was successfully eliminated**.

Exhibit 3: Auto-correlation function of the residuals of a fitted ARMA(1,1) model.
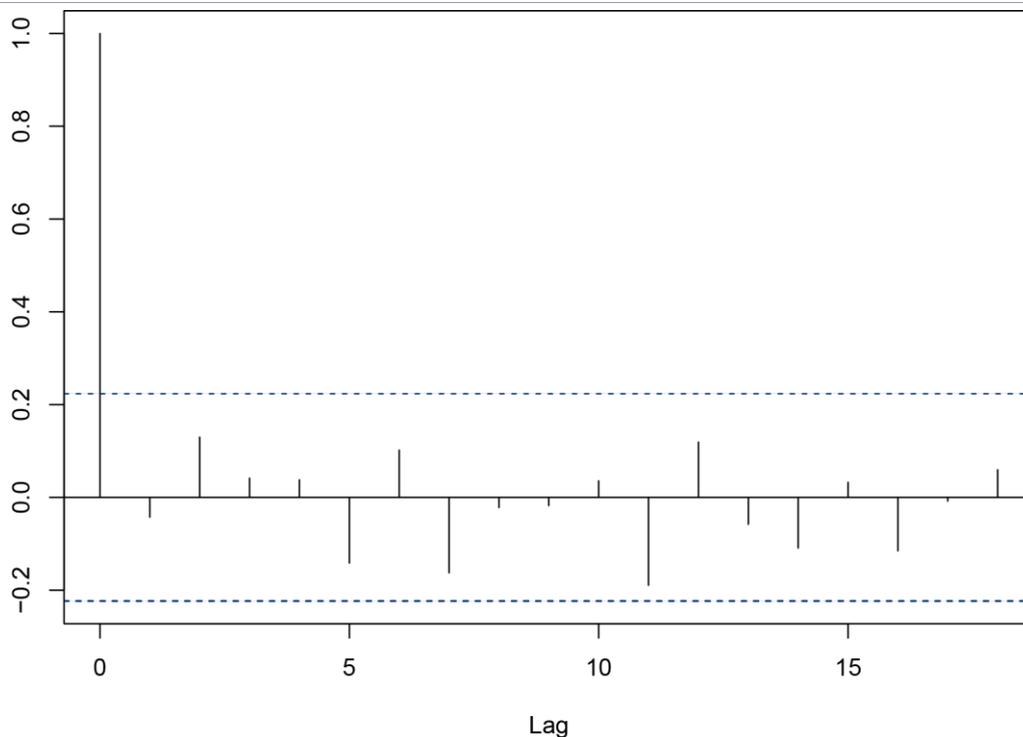
Source: Analysis based on Thomson Reuters data. The dashed lines show the 95%-confidence intervals.

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[6] Mathematically speaking an ARMA process is defined as follows: given an i.i.d. noise process (Z_t), $Z_t \sim F$ with mean zero, the ARMA(p,q) process is written as: $X_t = \phi_1 X_{t-1} + .. + \phi_p X_{t-p} + \theta_1 Z_{t-1} + .. + \theta_q Z_{t-q} + Z_t$.

Private equity under Solvency II:
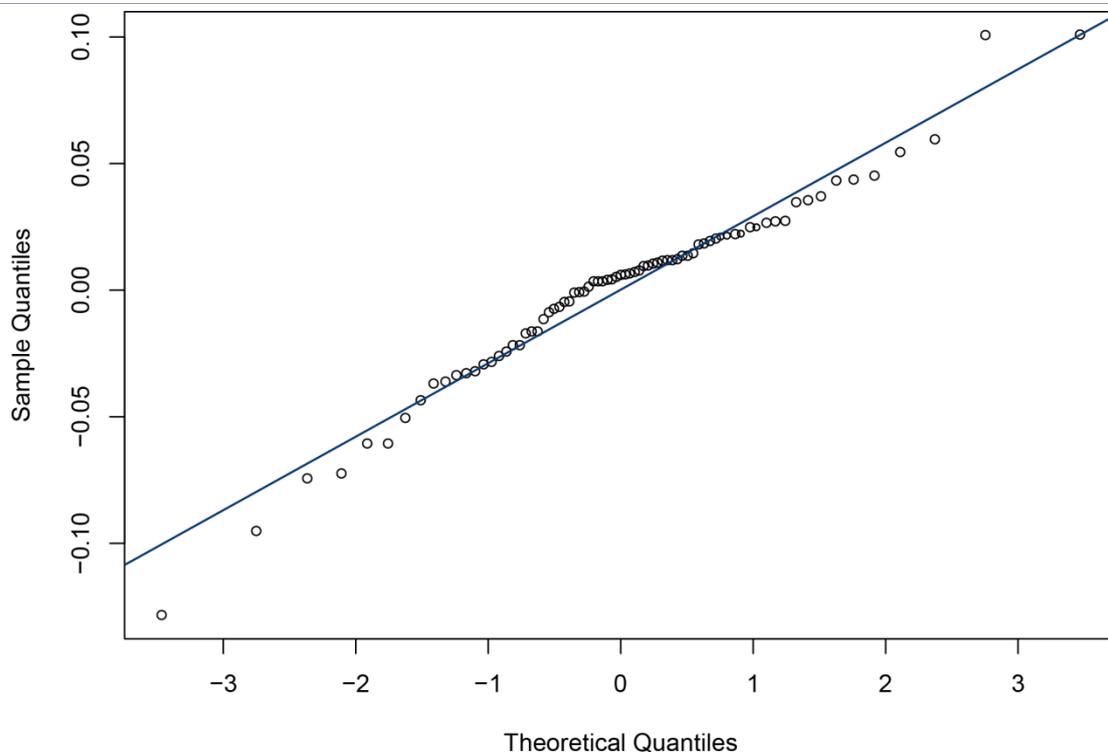# Evidence from time series models

The assumed distribution of the noise variable can be tested with so-called quantile-quantile plots, or simply q-q-plot. Typically, observations from financial time series have heavier tails than a normal distribution suggests. Exhibit 4 gives a q-q plot for the noise variables of the estimated ARMA(1,1) model, where the distribution of the noise term is supposed to be a student t distribution with *df*=4. Significant outliers and a shape of the points differing significantly from a straight line would provide evidence against the assumed model. Because **the model provides a good fit, we will stick to ARMA models to estimate the one-year 99.5% VaR** that is needed under Solvency II.

In order to get models, that can be further analyzed mathematically, the coefficients of the ARMA models have to fulfill certain constraints. These constraints enable us to write $X_{t+1} = \mu_{t+1} + Z_{t+1}$ at time t, where the term μ_{t+1} is known at time t and Z_{t+1} is independent of the information up to time t. In contrast to the typical random walk process, the future development of ARMA processes is dependent on the observable information at any given point in time t. Denoting the VaR conditional on the information up to time t by VaR^t we may write

$$VaR^t(X_{t+1}) = \mu_{t+1} + VaR^t(Z_{t+1}) = \mu_{t+1} + VaR(Z_{t+1}),$$

where we use the fact that the term μ_{t+1} is known at time t and can therefore be treated as a constant. Furthermore Z_{t+1} is independent of the information up time t such that conditional VaR^t and unconditional VaR are equal. This equation allows us to calculate a VaR for private equity returns.

**Exhibit 4: q-q-plot of the noise term in the ARMA(1,1) model against a student t distribution with *df*=4.**

Source: Analysis based on Thomson Reuters data.

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Next to the graphical illustration of the fit of an ARMA model to private equity data, we also aim at backtracking the calculated VaR using ARMA models with the observed data. Let n be the number of observations on which we can test the number of breaches of the VaR^t, at a confidence level α (e.g. for a 99.5% VaR). In a well fitted model there should be about $n(1 - \alpha)$ breaches. Solvency II asks for capital requirements equivalent to the 99.5% quantile: with private equity giving basically a 20-year history of quarterly data, it is statistically challenging to backtest the model VaR with actual breaches observed in the data.

Exhibit 5 shows the backtesting for the ARMA(1,1) model for the quarterly log-returns showing the private equity return series as well as the 99.5%- and 0.5%-quantiles for the normal and the t distribution. The blue line shows the VaR^t under the assumption that the noise component is normally distributed and the blue line shows the result under the assumption that the noise component follows a scaled student t distribution. In both cases we show the upper $VaR^t_{0.995}$ and the lower $VaR^t_{0.005}$. We are testing it for $n = 32$ time points such that there should be around 0.16 breaches (each actual breach is marked with a dot)[7].

Exhibit 5: Backtesting for the log-returns in the ARMA(1,1) model for pe.

Source: Analysis based on Thomson Reuters data.
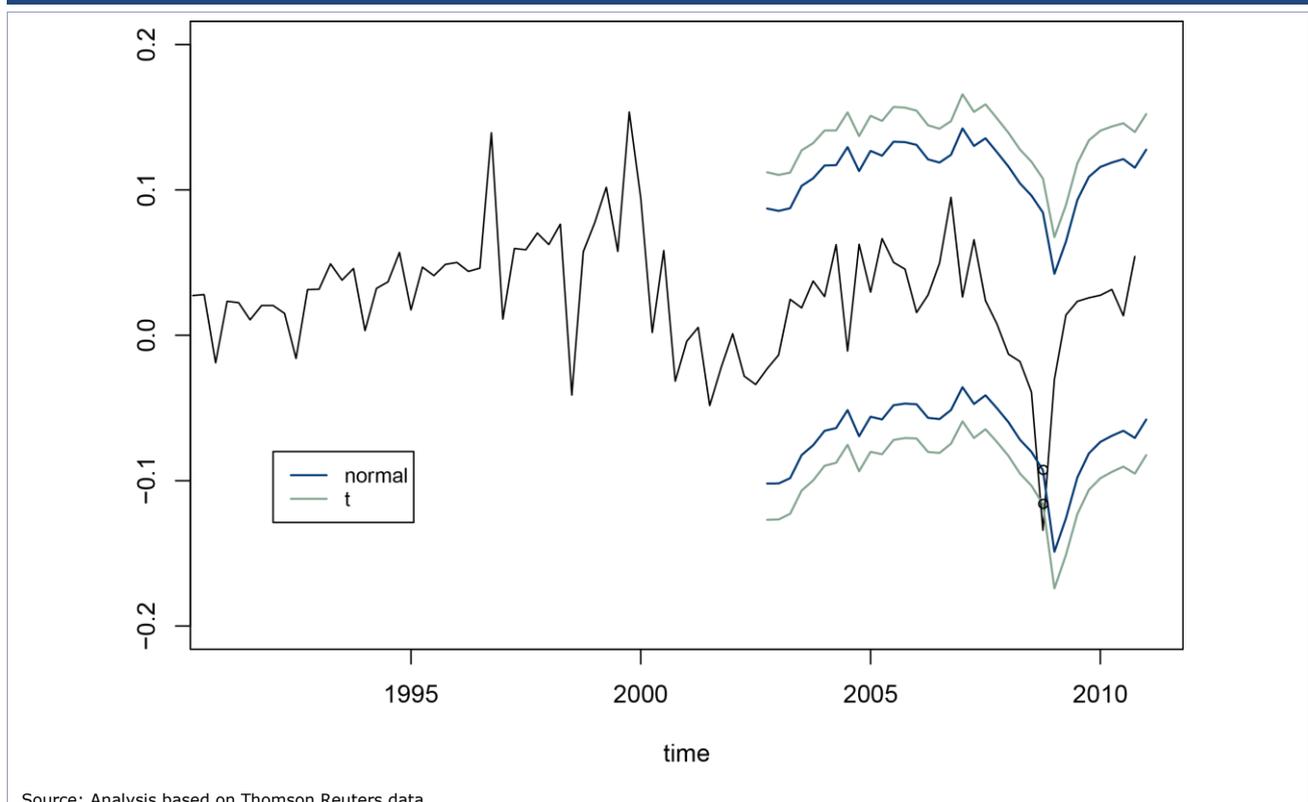
There is only one breach of investigated quantiles during the last financial crisis. To avoid this breach, an $\alpha \leq 6 \cdot 10^{-5}$ would be required in the case of the normal distribution. The occurrence of such a rare event, under the assumption of the normal distribution, can be interpreted as evidence against the normal distribution. However, under the assumption of the

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[7] In the referenced study breaches for $\alpha = 0.7$, $\alpha = 0.3$ respectively were also analyzed with an expected number of about 9.6 breaches.

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scaled t distribution, the breach is avoided with an $\alpha \leq 0.0027$. In our opinion, this is acceptable when considering the magnitude of the financial crisis.

As we have shown above, the yearly returns can be written in terms of the quarterly log-returns as follows:

$$RY_{t+4} = \exp(X_{t+1} + X_{t+2} + X_{t+3} + X_{t+4}) - 1.$$

At time t we are interested in estimating the $VaR_{0.005}^{t}$ of the yearly return RY_{t+4}. To this end it is possible to write

$$X_{t+1} + X_{t+2} + X_{t+3} + X_{t+4} = \mu_{t,4} + Z_{t,4},$$

where $\mu_{t,4}$ is known at time t and $Z_{t,4}$ is an expression that is independent of the information up to time t. Now we can calculate the yearly $VaR_{0.005}^{t}$ at time t by

$$VaR_{0.005}^{t}(RY_{t+4}) = \exp(\mu_{t,4} + VaR(Z_{t,4})) - 1 .$$

We have calculated the annual VaR for AR(2) and ARMA(1, 1) models from 2002 to 2010. The minima, maxima and average of these calculations are given in Exhibit 6. The calculations are performed such that at time t we do not use any future information. The coefficients of the models are estimated with the data given up to time t. As a result, we have observed (not shown here) that the dynamically estimated coefficients of the AR and ARMA models do not vary strongly over time, which further underlines the good fit and stability of the model. The orders $p = 2$ or $(p, q) = (1, 1)$ of the AR and the ARMA models where chosen to be optimal by a version of the Akaike Information Criterion, which is standard practice in estimating the order of ARMA models. The noise variables are assumed to have a normal distribution and a scaled student t distribution with degrees of freedom $df = 4$. The scaled t distribution has shown to provide a better fit with the data.

Exhibit 6: Minima, maxima and average of the yearly $VaR_{0.005}^{t}(RY_{t+4})$.

	t distribution		
data	Max	Min	Ø
pe^1	-0.09	-0.40	-0.19
pe^2	-0.09	-0.39	-0.19

[1]AR(2) model, [2]ARMA(1,1) model

Data source: Thomson Reuters

In the standard model of Solvency II, a symmetric adjustment mechanism to the VaR^t is applied in order to prevent firesales and procyclical effects. This adjustment was also analyzed. When allowing, as in the standard model, the symmetric adjustment mechanism to maximally change the calculated VaR^t by ± 0.1, **stress factors for private equity ranged between 12% to 30%** over the observed time period **suggesting a maximum stress factor of not more than 30%**.

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AGGREGATION OF PRIVATE EQUITY WITH GLOBAL EQUITY

Next to the estimation of the suitable stress factors insurance companies will need to aggregate private equity within the Solvency II framework to calculate an aggregate stress factor. In line with the application of ARMA models in the one-dimensional case, the quarterly log-returns of *pe* and of the MSCI were modeled as a bivariate vector AR(p) process. For a vector AR(p) process, the realization of one component is the sum of linear combinations of the components of the past p realizations and a noise term. In practice multivariate AR(p) models are generally preferred to multivariate ARMA(p,q) models. We restrict our further analyses to the bivariate vector AR(1) model, which gives a more stable fit than a vector AR(2) model.

We assume that the first component of a bivariate vector AR(1) process are the log-returns of *pe* and the second component are the log-returns of the MSCI. The log-return of *pe* at time t+1 is thus a linear combination of the log-returns of *pe* and MSCI at time t plus a noise term. The noise component is assumed to have a bivariate t distribution.

We consider a portfolio of private equity and global equity with an overall investment volume of 1 unit. The private equity ratio is denoted as γ. Let $RY_{t+4,pe}$ and $RY_{t+4,MSCI}$ be the simple yearly returns of *pe* and MSCI. The simple yearly return of a portfolio with private equity ratio γ is given by

$$RY_{t+4} = \gamma RY_{t+4,\text{pe}} + (1-\gamma)RY_{t+4,\text{MSCI}}.$$

At time t we are interested in estimating

$$VaR^t_{0.005} = VaR^t_{0.005}\big(\gamma RY_{t+4,\text{pe}} + (1-\gamma)RY_{t+4,\text{MSCI}}\big).$$

Let $X_{t,pe}$ and $X_{t,MSCI}$ denote the log-returns of *pe* and MSCI. $VaR^t_{0.005}$ can be written as follows

$$VaR^t_{0.005}\Big(\gamma\big(\exp(X_{t+1,pe} + \cdots + X_{t+4,pe}) - 1\big) + (1-\gamma)\big(\exp(X_{t+1,MSCI} + \cdots + X_{t+4,MSCI}) - 1\big)\Big).$$
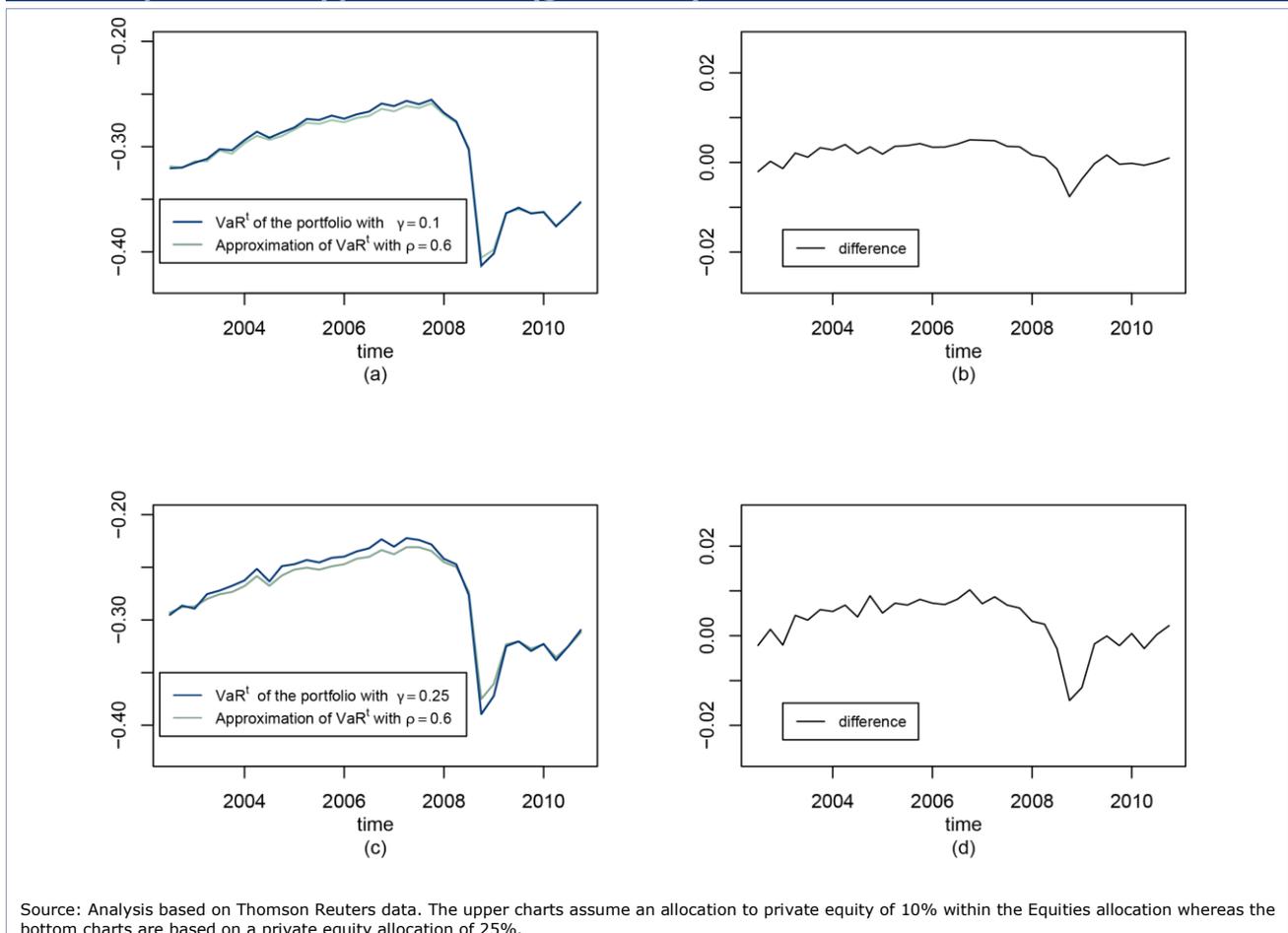
Assuming that the log-returns $(X_{t,pe}, X_{t,MSCI})$ follow a bivariate AR(1) model, the yearly $VaR^t_{0.005}$ of the portfolio can now be calculated for a fixed private equity allocation by means of simulations. This result can then be compared to the result of aggregating $\gamma VaR^t_{0.005}(RY_{t+4,\text{pe}})$ and $(1-\gamma)VaR^t_{0.005}(RY_{t+4,\text{MSCI}})$ applying the square root principle.

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Exhibit 7 shows the approximation using an aggregation coefficient of 0.6 for an allocation to private equity ratio of 10% (upper charts) or 25% (lower charts). The difference between the estimated $VaR^t_{0.005}$ of the portfolio and the square root approximation is given in (b) and (d). We conclude that the **aggregation coefficient ρ = 0.6 provides a good approximation**. This compares favorably to the 0.75 that are requested under the standard model and increases the diversification benefits for insurance companies.

Exhibit 7: $VaR^t_{0.005}$ of the yearly returns RY_{t+4} in a bivariate AR(1) model (blue line) with square-root approximation (green line).

Source: Analysis based on Thomson Reuters data. The upper charts assume an allocation to private equity of 10% within the Equities allocation whereas the bottom charts are based on a private equity allocation of 25%.

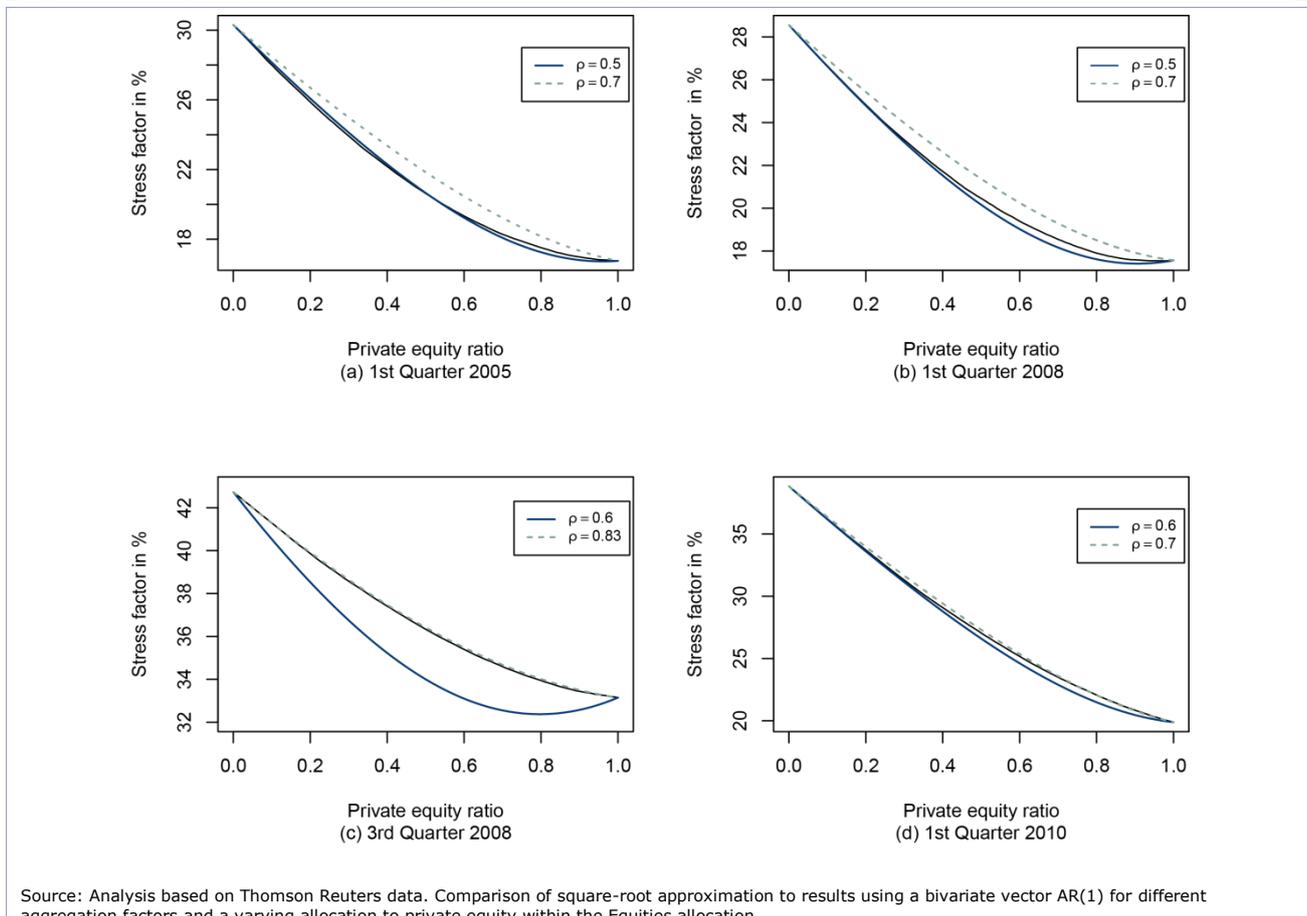Partners Group
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As a final analysis, the development of a suitable aggregation coefficient over time is analyzed. We fix the time t to the 1st quarter of 2005, the 1st quarter of 2008, the 3rd quarter of 2008 and the 1st quarter of 2010. We then compare the estimated $VaR^t_{0.005}$ of the portfolio and the aggregated result for a varying private equity ratio $\gamma \in [0, 1]$. To be more precise we show $VaR^t_{0.005}$ in percent (black line) for varying γ. The result is shown in Exhibit 8, where for each considered quarter the square root approximations are again compared to the simulated portfolio $VaR^t_{0.005}$.

It turns out that during the last financial crisis an aggregation coefficient of 0.83 would have been a good approximation, but for the rest of the time this aggregation coefficient is too high. Nevertheless the error made when using $\rho = 0.6$ instead of 0.83 in the 3rd quarter of 2008 would have been smaller than 2% for a private equity ratio $\gamma \leq 0.3$.

Exhibit 8: Stress factor for a portfolio in a bivariate vector AR(1) model for varying private equity ratio with square-root approximation.

Source: Analysis based on Thomson Reuters data. Comparison of square-root approximation to results using a bivariate vector AR(1) for different aggregation factors and a varying allocation to private equity within the Equities allocation.

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CONCLUSION

With Solvency II coming into effect at the beginning of 2013, insurance companies across Europe are in the process of finding their way through the additional complexity that will be introduced by this new regulation. Next to using the standard model, insurance companies may use full or partial internal models to calculate their solvency capital.

The currently proposed stress factor for private equity of 49% can in our opinion not be backed by actual data from unlisted private equity. In contrast, using time series models it is possible to show that the requested 99.5% VaR for private equity is considerably lower and **does not exceed 30% over the observed time period** when a symmetric dampener is taken into consideration. This information is highly relevant for insurance companies that may consider using (partial) internal models, which is a significant part of the European insurance landscape (over 90% of the respective respondents answered that they were currently working on the implementation of their internal model for Solvency II purposes[8] during the most recent quantitative impact study). Next to the asset class specific stress factor, an insurance company will need to determine how to aggregate stress factors across different asset classes. Results indicate that an **aggregation coefficient of 0.6 provides a good approximation**.

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[8] *EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II*, European Insurance and Occupational Pensions Authority (EIOPA), March 2011

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